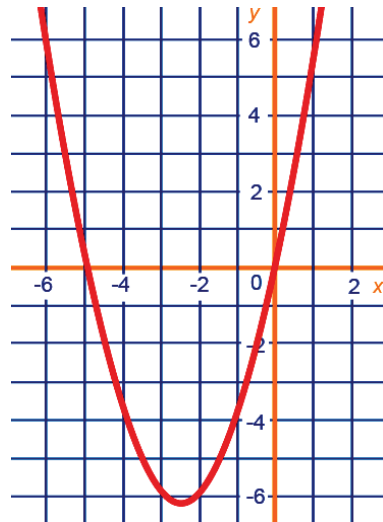
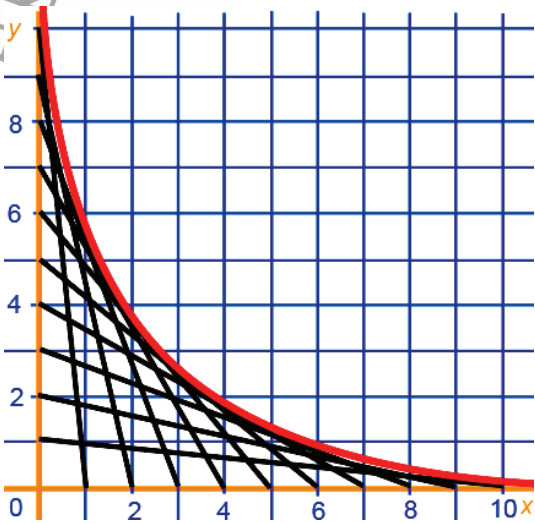


Hoofdstuk 29 PARABOLEN

29.0 INTRO

1 ab



c ...

d $x = -2\frac{1}{2}$

4 $(3,9); (-2\frac{1}{2}, -6)$

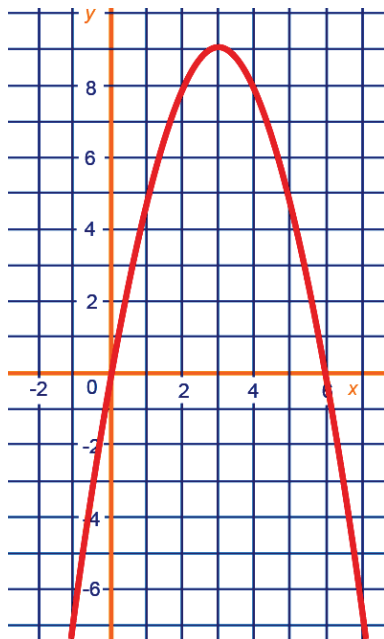
29.1 PARABOLEN

2 a $6 - x$

b $y = x(6 - x)$

c

x	-1	0	1	2	3	4	5	6	7
y	-7	0	5	8	9	8	5	0	-7



d ...

e $x = 3$

3 a $y = x(x + 5)$

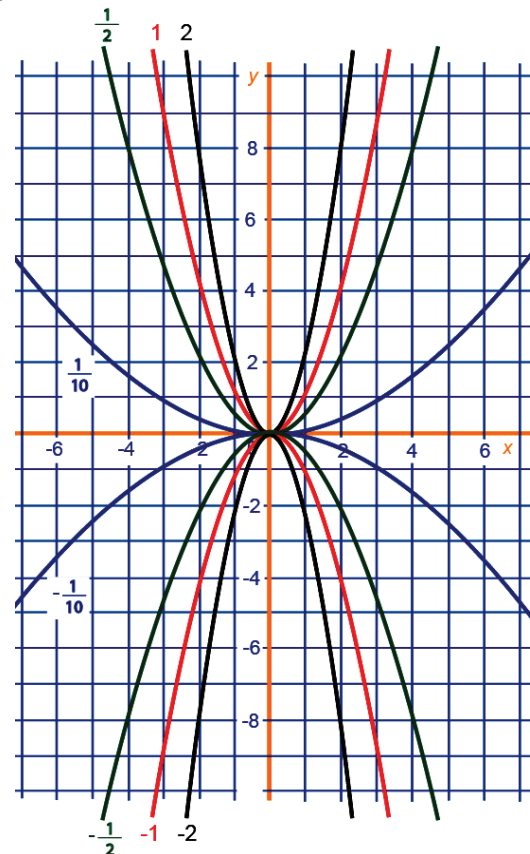
b

x	-6	-5	-4	-3	-2	-1	0	1	2
y	6	0	-4	-6	-6	-4	0	6	14

5 a

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = \frac{1}{10}x^2$	0,9	0,4	0,1	0	0,1	0,4	0,9
$y = \frac{1}{2}x^2$	4,5	2	0,5	0	0,5	2	4,5
$y = 2x^2$	18	8	2	0	2	8	18

bd



c

x	-3	-2	-1	0	1	2	3
$y = -x^2$	-9	-4	-1	0	-1	-4	-9
$y = -\frac{1}{10}x^2$	-0,9	-0,4	-0,1	0	-0,1	-0,4	-0,9
$y = -\frac{1}{2}x^2$	-4,5	-2	-0,5	0	-0,5	-2	-4,5
$y = -2x^2$	-18	-8	-2	0	-2	-8	-18

- e Dalparabool als $c > 0$,
een bergparabool als $c < 0$.
- f Ze zijn elkaars spiegelbeeld in de x-as.
- g Dan is $y = 0$, dat is een rechte lijn, dat is de vergelijking van de x-as.

- 6 $y = cx^2$
 $3 = c \cdot 1^2$ (invullen het punt (1,3))
 $3 = c$
- $y = cx^2$
 $2 = c \cdot (-5)^2$ (invullen het punt (-5,2))
 $2 = 25c$
 $\frac{2}{25} = c$
- $y = cx^2$
 $-3 = c \cdot 3^2$ (invullen het punt (3,-3))
 $-3 = 9c$
 $-\frac{1}{3} = c$
- 7 $y = cx^2$
 $4 = c \cdot 5^2$ (invullen het punt (5,4) of (-5,4))
 $4 = 25c$
 $\frac{4}{25} = c$

29.2 HERHALING

- 8 $12 - 18x$
 $8x^2 - 20x$
 $-4x + 10x^2$
 $3x - 8$
 $-2x^2 - 39x - 10$
 $15x^2 - 9x - 13$
 $22 + 2x^2 - 6x$
 $-3x + 4y$
 $5x + 3y$
- 9 $x^2 - 10x + 21$
 $6x^2 + 22x - 8$
 $p^2 + 6p + 9$
 $-p^2 + 4q^2$
- $3x^2 + 5x - 8$
 $2x^2 + 3x - 2$
 $25 - 10q + q^2$
 $4p^2 - 12pq + 9q^2$

- 10 $x(x+7)$ $(x-3)^2$
 $x(x-10)$ $(x+5)^2$
 $(x-7)(x-1)$ $(x-6)^2$
 $(x-3)(x+9)$ $(2x-3)^2$

- 11 $x^2 + 10x = -16$ ➤ PLUS 16
 $x^2 + 10x + 16 = 0$ ➤ ONTBINDEN
 $(x+2)(x+8) = 0$
 $x = -2$ of $x = -8$

- $10x = x^2$ ➤ MIN x^2
 $10x - x^2 = 0$ ➤ ONTBINDEN
 $x(10-x) = 0$
 $x = 0$ of $x = 10$

- $x^2 + 6x = 16$ ➤ MIN 16
 $x^2 + 6x - 16 = 0$ ➤ ONTBINDEN
 $(x+8)(x-2) = 0$
 $x = -8$ of $x = 2$

- $x^2 + 16 = 8x$ ➤ MIN $8x$
 $x^2 - 8x + 16 = 0$ ➤ ONTBINDEN
 $(x-4)^2 = 0$
 $x = 4$

- $3(x+1) = x^2 + 5$ ➤ HAAKJES WEG
 $3x + 3 = x^2 + 5$ ➤ MIN $3x$, MIN 3
 $x^2 - 3x + 2 = 0$ ➤ ONTBINDEN
 $(x-2)(x-1) = 0$
 $x = 2$ of $x = 1$

- $(x+1)(x+3) = 1 - x^2$ ➤ HAAKJES WEG
 $x^2 + 4x + 3 = 1 - x^2$ ➤ PLUS x^2 , MIN 1
 $2x^2 + 4x + 2 = 0$ ➤ DELEN DOOR 2
 $x^2 + 2x + 1 = 0$ ➤ ONTBINDEN
 $(x+1)(x+1) = 0$
 $x = -1$

- $x^2 - 5x = 6$ ➤ MIN 6
 $x^2 - 5x - 6 = 0$ ➤ ONTBINDEN
 $(x-6)(x+1) = 0$
 $x = 6$ of $x = -1$

- $3 - 4x = 1 - 2x^2$ ➤ PLUS $2x^2$, MIN 1
 $2x^2 - 4x + 2 = 0$ ➤ DELEN DOOR 2
 $x^2 - 2x + 1 = 0$ ➤ ONTBINDEN
 $(x-1)(x-1) = 0$
 $x = 1$

- $12 - 11x = x^2$ ➤ PLUS $11x$, MIN 12
 $x^2 + 11x - 12 = 0$ ➤ ONTBINDEN
 $(x+12)(x-1) = 0$
 $x = -12$ of $x = 1$

- $3x^2 = 6x - 3$ ➤ MIN $6x$, PLUS 3
 $3x^2 - 6x + 3 = 0$ ➤ DELEN DOOR 3
 $x^2 - 2x + 1 = 0$ ➤ ONTBINDEN
 $(x-1)(x-1) = 0$
 $x = 1$

$$5x^2 = -15x$$

$$5x^2 + 15x = 0$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0 \text{ of } x = 3$$

- PLUS 15x
- DELEN DOOR 5
- ONTBINDEN

$$2(x^2 - 2) = 4(x^2 - 3)$$

$$2x^2 - 4 = 4x^2 - 12$$

$$2x^2 - 8 = 0$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x = 2 \text{ of } x = -2$$

- HAAKJES WEG
- MIN $2x^2$, PLUS 4
- DELEN DOOR 2
- ONTBINDEN

29.3 PARABOLEN TEKENEN

- 12 a** $h = 0,0625 \cdot 40^2 = 100$ m
- b** als $x = 0$, $h = 0,0625 \cdot 0^2 = 0$ m
als $x = 10$, $h = 0,0625 \cdot 10^2 = 6,25$ m
als $x = 20$, $h = 0,0625 \cdot 20^2 = 25$ m
als $x = 30$, $h = 0,0625 \cdot 30^2 = 56,25$ m
als $x = 40$, $h = 0,0625 \cdot 40^2 = 100$ m
- c** $x = 35$, dan $h = 0,0625 \cdot 35^2 = 76,5625$ m
De hoogte boven de Wupper is dan
 $100 - 76,5625 = 23,4375$ m

- 13 a** $x^2 - 6x = 0$
 $(x-6) = 0$
 $x = 0$ of $x = 6$
Dus de nulpunten zijn 0 en 6.

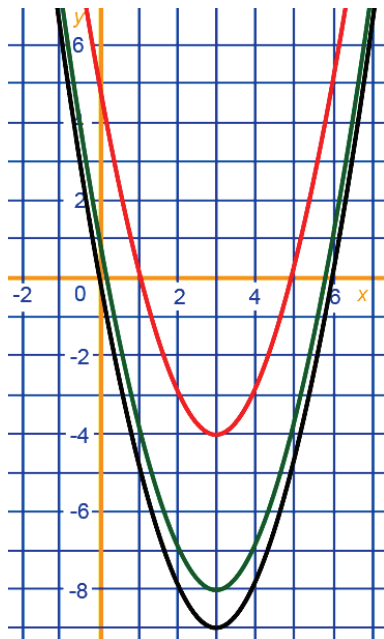
b symmetrie-as: $x = \frac{0+6}{2} = 3$

c $y = 3^2 - 6 \cdot 3 = -9$, Top(3,-9)

d

x	-1	0	$\frac{1}{2}$	1	2	3	4	5	$5\frac{1}{2}$	6	7
y	7	0	$-2\frac{3}{4}$	-5	-8	-9	-8	-5	$-2\frac{3}{4}$	0	7

ef



g 9

h $y = x^2 - 6x + 9 = (x-3)^2$

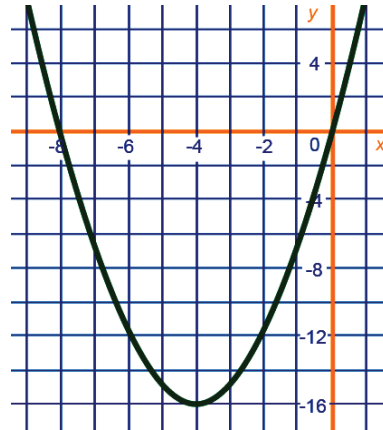
- 14 a** $x^2 + 8x = 0$
 $x(x+8) = 0$
 $x = 0$ of $x = -8$
Dus de nulpunten zijn 0 en -8.

b symmetrie-as: $x = \frac{-8+0}{2} = -4$

c $y = (-4)^2 + 8 \cdot (-4) = -16$
Top(-4,-16)

d

x	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
y	9	0	-7	-12	-15	-16	-15	-12	-7	0	9



e 16

f $y = x^2 + 8x + 16 = (x+4)^2$

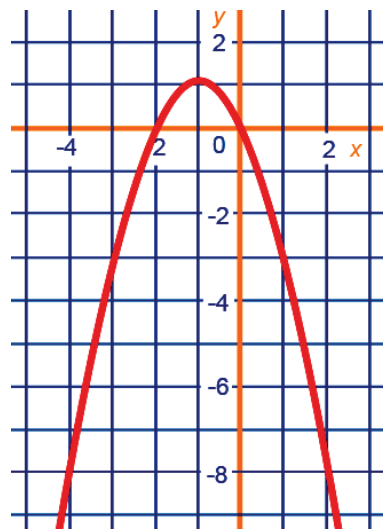
- 15 a** $-x^2 - 2x = 0$
 $-x(x+2) = 0$
 $x = 0$ of $x = -2$
Dus de nulpunten zijn 0 en -2.

b symmetrie-as: $x = \frac{-2+0}{2} = -1$

c $y = (-1)^2 - 2 \cdot (-1) = -1 + 2 = 1$
Top(-1,1)

d

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y	-24	-15	-8	-3	0	1	0	-3	-8	-15	-24



e -1

16 a snijpunt y-as $\Rightarrow x = 0$

$$y = 0^2 - 2 \cdot 0 + 4 = 4$$

(0,4)

b $x^2 - 2x + 4 = 4$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0 \quad \text{of} \quad x = 2$$

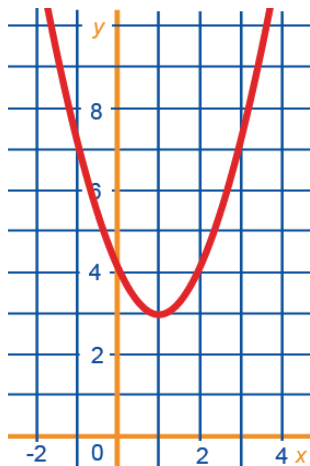
c symmetrie-as: $x = \frac{0+2}{2} = 1$

d $y = 1^2 - 2 \cdot 1 + 4 = 3$

Top(1,3)

e

x	-2	-1	0	1	2	3	4
y	12	7	4	3	4	7	12



17 a $-2 \cdot 0^2 + 4 \cdot 0 + 2 = 2$

Snijpunt y-as (0,2)

b $-2x^2 + 4x + 2 = 2$

$$-2x^2 + 4x = 0$$

$$-2x(x - 2) = 0$$

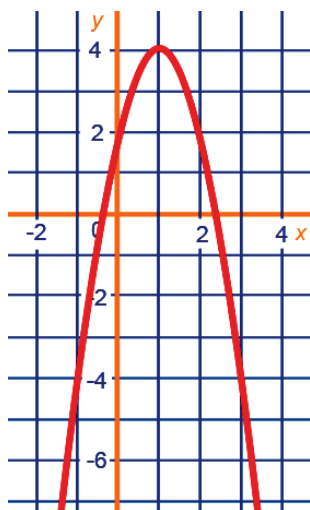
$$x = 0 \quad \text{of} \quad x = 2$$

c symmetrie-as: $x = \frac{0+2}{2} = 1$

d $y = -2 \cdot 1^2 + 4 \cdot 1 + 2 = 4$

Top(1,4)

e



18 $y = x^2 - 2x$

Nulpunten :

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0 \quad \text{of} \quad x = 2$$

Snijpunt y - as :

$$y = 0^2 + 2 \cdot 0 = 0$$

(0,0)

Symmetrie - as :

$$x = \frac{0+2}{2} = 1$$

$$y = 1^2 - 2 \cdot 1 = -1$$

Top(1,-1)

x	-2	-1	0	1	2	3	4
y	8	3	0	-1	0	3	8

$y = -x^2 + 5x - 2$

Snijpunt y - as :

$$y = -0^2 + 5 \cdot 0 - 2 = -2$$

(0,-2)

$$-x^2 + 5x - 2 = -2$$

$$-x^2 + 5x = 0$$

$$-x(x - 5) = 0$$

$$x = 0 \quad \text{of} \quad x = 5$$

Symmetrie - as :

$$x = \frac{0+5}{2} = 2\frac{1}{2}$$

$$y = -(2\frac{1}{2})^2 + 5 \cdot 2\frac{1}{2} - 2 = 4\frac{1}{4}$$

Top(2 $\frac{1}{2}$, 4 $\frac{1}{4}$)

x	-1	0	1	2	2 $\frac{1}{2}$	3	4	5	6
y	-8	-2	2	4	4 $\frac{1}{4}$	4	2	-2	-8

$y = (x + 2)(x - 8)$

Nulpunten :

$$(x + 2)(x - 8) = 0$$

$$x = -2 \quad \text{of} \quad x = 8$$

Snijpunt y - as :

$$y = (0 + 2)(0 - 8) = -16$$

(0,-16)

Symmetrie - as :

$$x = \frac{-2+8}{2} = 3$$

$$y = (3 + 2)(3 - 8) = -25$$

Top(3,-25)

x	0	1	2	3	4	5	6
y	-16	-21	-24	-25	-24	-21	-16

$$y = 2x^2 - 3x + 1$$

Snijpunt y – as :

$$y = 2 \cdot 0^2 - 3 \cdot 0 + 1 = 1$$

(0,1)

$$2x^2 - 3x + 1 = 1$$

$$2x^2 - 3x = 0$$

$$2x(x - 1\frac{1}{2}) = 0$$

$$x = 0 \quad \text{of} \quad x = 1\frac{1}{2}$$

Symmetrie – as :

$$x = \frac{0+1\frac{1}{2}}{2} = \frac{3}{4}$$

$$y = 2 \cdot (\frac{3}{4})^2 - 3 \cdot \frac{3}{4} + 1 = -\frac{1}{8}$$

Top($\frac{3}{4}, -\frac{1}{8}$)

x	-2	-1	0	$\frac{3}{4}$	1	2	3
y	15	6	1	$-\frac{1}{8}$	0	3	10

$$y = -3x^2 - 9x + 6$$

Nulpunten :

$$-3x^2 - 9x + 6 = 0$$

$$x^2 + 3x + 2 = 0$$

$$(x+1)(x+2) = 0$$

$$x = -1 \quad \text{of} \quad x = -2$$

Snijpunt y – as :

$$y = -3 \cdot 0^2 - 9 \cdot 0 + 6 = 6$$

(0,6)

Symmetrie – as :

$$x = \frac{-1-2}{2} = -1\frac{1}{2}$$

$$y = -3 \cdot (-1\frac{1}{2})^2 - 9 \cdot -1\frac{1}{2} + 6 = 12\frac{3}{4}$$

Top($-1\frac{1}{2}, 12\frac{3}{4}$)

x	-4	-3	-2	$-1\frac{1}{2}$	-1	0	1
y	-6	6	12	$12\frac{3}{4}$	12	6	-6

$$y = x(x-8) + 2$$

Snijpunt y – as :

$$y = 0 \cdot (0-8) + 2 = 2$$

(0,2)

$$x(x-8) + 2 = 2$$

$$x(x-8) = 0$$

$$x = 0 \quad \text{of} \quad x = 8$$

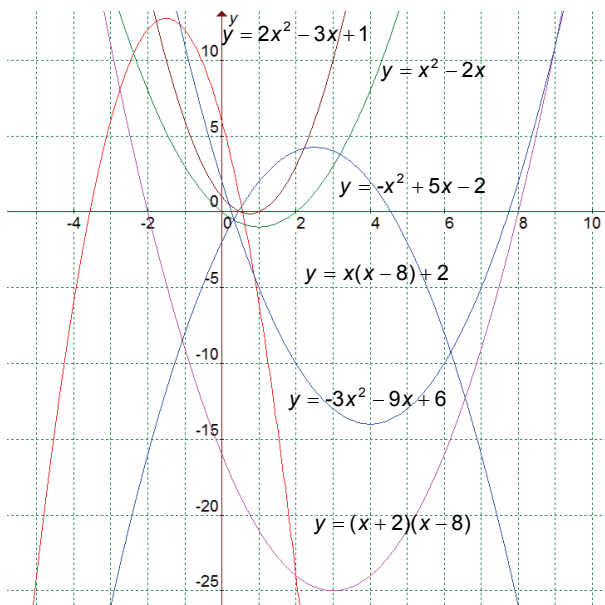
symmetrie – as :

$$x = \frac{0+8}{2} = 4$$

$$y = 4 \cdot (4-8) + 2 = -14$$

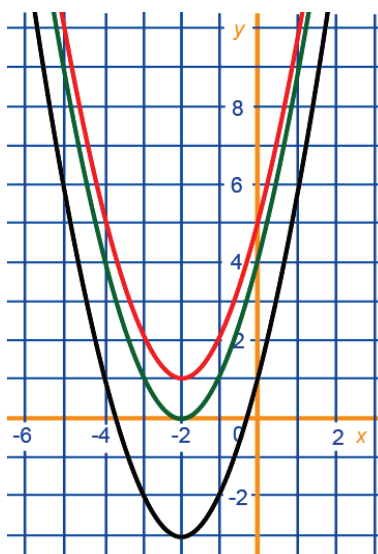
Top(4, -14)

x	-1	0	1	2	3	4	5	6	7
y	11	2	-5	-10	-13	-14	-13	-10	-5

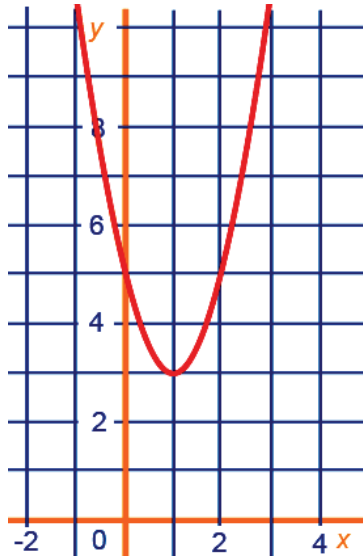


- 19 a (0,0)
b (0,1) ; (0,-3)
c $y \geq 0$
d (-2,0)
e (-2,1) ; (-2,-3)
f

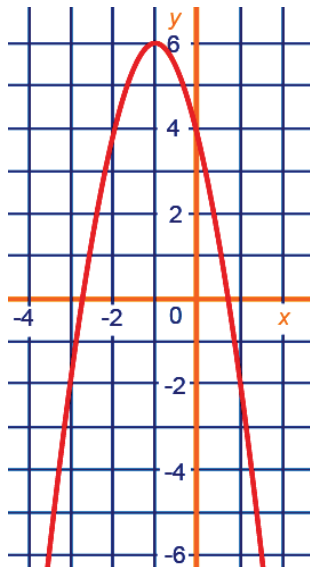
x	-5	-4	-3	-2	-1	0	1
$y = (x+2)^2$	9	4	1	0	1	4	9
$y = (x+2)^2 + 1$	10	5	2	1	2	5	10
$y = (x+2)^2 - 3$	6	1	-2	-3	-2	1	6



- 20 a** Omdat $2(x-1)^2$ voor elke waarde van x , 0 of meer is. Dus $2(x-1)^2 + 3$ is minimaal 3.
b $y \geq 3$
c een dalparabool, omdat $2(x-1)^2$ minimaal 0 is
d bijv. $y = 2(x-1)^2$
e door bijv. van $2(x-1)^2$ een getal af te trekken
f symmetrie-as: $x = 1$
g Top(1,3)
h



- 21 a** een bergparabool, omdat $-2(x+1)^2$ maximaal 0 is
b twee nulpunten, omdat de top op hoogte 6 ligt en het een bergparabool is
c symmetrie-as: $x = -1$
d Top(-1,6)
e



- 22 a** Dalparabool als $c > 0$,
 een bergparabool als $c < 0$.
b (a,b)
23 (-2,-3) en (9,19)

29.4 TOEPASSINGEN

- 24 a** $x - \frac{1}{100}x^2 = 0$
 $\frac{1}{100}x(100-x) = 0$
 $x = 0$ of $x = 100$
 Dus de kogel legt 100 meter af.
b Vanwege symmetrie wordt de grootste hoogte bereikt als $x = 50$.
 Dan $y = 50 - \frac{1}{100} \cdot 50^2 = 25$, dus 25 meter.

- 25 a** $C = 3 \cdot 1\frac{1}{2} \cdot 10 = 45$
b $C = x(6-2x) \cdot 10 = 60x - 20x^2$
c een bergparabool
d Nulpunten:
 $60x - 20x^2 = 0$
 $20x(3-x) = 0$
 $x = 0$ of $x = 3$
 Top ligt bij $x = 1\frac{1}{2} \Rightarrow y = 60 \cdot 1\frac{1}{2} - 2 \cdot (1\frac{1}{2})^2 = 45$
 Top($1\frac{1}{2}$, 45)
e bij hoogte $1\frac{1}{2}$, de capaciteit is dan 45

- 26** $y = cx^2$
 $62,5 = c \cdot 250^2$ (invullen het punt (250;62,5))
 $62,5 = 62500c$
 $\frac{1}{1000} = c$
 Vergelijking parabool: $y = \frac{1}{1000}x^2$

- 27 a** $(10+2)^2 - 10 - 10 = 124$ stippen
b $(n+2)^2 - 2n = n^2 + 2n + 4$
c $n^2 + 2n + 4 = 10204$
 $n^2 + 2n - 10200 = 0$
 $(n-100)(n+102) = 0$
 $n = 100$ of $n = -102$
 Alleen $n = 100$ voldoet.
d $n^2 + 2n + 4 + 43 = (n+1)^2 + 2(n+1) + 4$
 $n^2 + 2n + 47 = n^2 + 4n + 7$
 $40 = 2n$
 $20 = n$
 Rangnummer is 20. Dus rangnummer 21 heeft 43 stippen meer dan rangnummer 20.

- 28 a** $(60-2x)^2 = 1000$
 $60-2x = \sqrt{1000}$ of $60-2x = -\sqrt{1000}$
 $2x = 60 - \sqrt{1000}$ of $2x = 60 + \sqrt{1000}$
 $2x = 60 - 10\sqrt{10}$ of $2x = 60 + 10\sqrt{10}$
 $x = 30 - 5\sqrt{10}$ of $x = 30 + 5\sqrt{10}$
 $x \approx 14,19$ cm of $x \approx 45,81$ cm
 Alleen $x = 30 - 5\sqrt{10} \approx 14,19$ cm voldoet.

b $(60 - 2x)^2 = 4 \cdot x \cdot (60 - 2x)$
 $3600 - 240x + 4x^2 = 240x - 8x^2$
 $12x^2 - 480x + 3600 = 0$
 $x^2 - 40x + 300 = 0$
 $(x - 30)(x - 10) = 0$
 $x = 30$ of $x = 10$
Alleen $x = 10$ cm voldoet.

29 a $-t^2 + 4t + 21 = 21$
 $-t^2 + 4t = 0$
 $-t(t - 4) = 0$
 $t = 0$ of $t = 4$
Dus na 4 sec. is het steentje weer op dezelfde hoogte.

b $-t^2 + 4t + 21 = 0$
 $t^2 - 4t - 21 = 0$
 $(t - 7)(t + 3) = 0$
 $t = 7$ of $t = -3$
Dus na 7 sec. bereikt het steentje het water.

c symmetrie-as: $t = \frac{7-3}{2} = 2$
 $h = -2^2 + 4 \cdot 2 + 21 = 25$
Op 2 sec. bereikt het steentje de maximale hoogte van 25 m.

30 a $2l + 2b = 44 \Rightarrow l + b = 22$

b $l + b = 22 \Rightarrow l = 22 - b$
 $(22 - b) \cdot b = 120$
 $22b - b^2 = 120$
 $b^2 - 22b + 120 = 0$
 $(b - 12)(b - 10) = 0$
 $b = 12$ of $b = 10$
 $l = 22 - 12 = 10$ $l = 22 - 10 = 12$
De rechthoek is 12 bij 10 of de rechthoek is 10 bij 12.

c $\begin{cases} l \cdot b = 720 \\ l = 5b \end{cases}$
 $5b \cdot b = 720$
 $5b^2 = 720$
 $b^2 = 144$
 $b = 12$ of $b = -12$
 $l = 5 \cdot 12 = 60$ (voldoet niet)
Dus de breedte is 12 en de lengte 60.

29.4 VERGELIJKINGEN OPLOSSEN

31 $(x - 3)^2 = 100$
 $x - 3 = 10$ of $x - 3 = -10$
 $x = 13$ of $x = -7$

$(x + \frac{1}{2})^2 = 25$
 $x + \frac{1}{2} = 5$ of $x + \frac{1}{2} = -5$
 $x = 4\frac{1}{2}$ of $x = -5\frac{1}{2}$

$(2x + 1)^2 = 36$
 $2x + 1 = 6$ of $2x + 1 = -6$
 $2x = 5$ of $2x = -7$
 $x = 2\frac{1}{2}$ of $x = -3\frac{1}{2}$

$(x - 3)^2 = 13$
 $x - 3 = \sqrt{13}$ of $x - 3 = -\sqrt{13}$
 $x = 3 + \sqrt{13}$ of $x = 3 - \sqrt{13}$

$(x + \frac{1}{2})^2 = 48$
 $x + \frac{1}{2} = \sqrt{48} = 4\sqrt{3}$ of $x + \frac{1}{2} = -\sqrt{48} = -4\sqrt{3}$
 $x = -\frac{1}{2} + 4\sqrt{3}$ of $x = -\frac{1}{2} - 4\sqrt{3}$

$(2x + 1)^2 = 68$
 $2x + 1 = \sqrt{68} = 2\sqrt{17}$ of $2x + 1 = -\sqrt{68} = -2\sqrt{17}$
 $2x = -1 + 2\sqrt{17}$ of $2x = -1 - 2\sqrt{17}$
 $x = -\frac{1}{2} + \sqrt{17}$ of $x = -\frac{1}{2} - \sqrt{17}$

32 25 ; 5
12 ; 36 ; x
16 ; x ; 8
 $20\frac{1}{4}$; $4\frac{1}{2}$
11 ; $30\frac{1}{4}$; x
7 ; x ; $3\frac{1}{2}$
4 ; 16
 $5\frac{1}{2}$; $30\frac{1}{4}$
3 ; 9

33 $x^2 + 10x = 90$
 $x^2 + 10x + 25 = 90 + 25$
 $(x + 5)^2 = 115$
 $x + 5 = \sqrt{115}$ of $x + 5 = -\sqrt{115}$
 $x = -5 + \sqrt{115}$ of $x = -5 - \sqrt{115}$

$$x^2 + 7x + 1 = 0$$

$$x^2 + 7x + 1 + 11\frac{1}{4} = 11\frac{1}{4}$$

$$x^2 + 7x + 12\frac{1}{4} = 11\frac{1}{4}$$

$$(x + 3\frac{1}{2})^2 = 11\frac{1}{4} = \frac{45}{4}$$

$$x + 3\frac{1}{2} = \sqrt{\frac{45}{4}} = \frac{1}{2}\sqrt{45} = 1\frac{1}{2}\sqrt{5} \quad \text{of}$$

$$x + 3\frac{1}{2} = -1\frac{1}{2}\sqrt{5}$$

$$x = -3\frac{1}{2} + 1\frac{1}{2}\sqrt{5} \quad \text{of} \quad x = -3\frac{1}{2} - 1\frac{1}{2}\sqrt{5}$$

$$x^2 + 10x + 22 = 0$$

$$x^2 + 10x + 22 + 78 = 78$$

$$x^2 + 10x + 100 = 78$$

$$(x + 10)^2 = 78$$

$$x + 10 = \sqrt{78} \quad \text{of} \quad x + 10 = -\sqrt{78}$$

$$x = -10 + \sqrt{78} \quad \text{of} \quad x = -10 - \sqrt{78}$$

$$x^2 - 11x = -7$$

$$x^2 - 11x + 30\frac{1}{2} = -7 + 30\frac{1}{2}$$

$$(x - 5\frac{1}{2})^2 = 23\frac{1}{2} = \frac{94}{4}$$

$$x - 5\frac{1}{2} = \sqrt{\frac{94}{4}} = \frac{1}{2}\sqrt{94} \quad \text{of} \quad x - 5\frac{1}{2} = -\frac{1}{2}\sqrt{94}$$

$$x = 5\frac{1}{2} + \frac{1}{2}\sqrt{94} \quad \text{of} \quad x = 5\frac{1}{2} - \frac{1}{2}\sqrt{94}$$

$$x^2 - 12x = -23$$

$$x^2 - 12x + 36 = -23 + 36$$

$$(x - 6)^2 = 13$$

$$x - 6 = \sqrt{13} \quad \text{of} \quad x - 6 = -\sqrt{13}$$

$$x = 6 + \sqrt{13} \quad \text{of} \quad x = 6 - \sqrt{13}$$

$$x^2 - 5x - 1 = 0$$

$$x^2 - 5x - 1 + 7\frac{1}{4} = 7\frac{1}{4}$$

$$x^2 - 5x + 6\frac{1}{4} = 7\frac{1}{4}$$

$$(x - 2\frac{1}{2})^2 = 7\frac{1}{4} = \frac{29}{4}$$

$$x - 2\frac{1}{2} = \sqrt{\frac{29}{4}} = \frac{1}{2}\sqrt{29} \quad \text{of} \quad x - 2\frac{1}{2} = -\frac{1}{2}\sqrt{29}$$

$$x = 2\frac{1}{2} + \frac{1}{2}\sqrt{29} \quad \text{of} \quad x = 2\frac{1}{2} - \frac{1}{2}\sqrt{29}$$

$$x^2 = x + 3$$

$$x^2 - x = 3$$

$$x^2 - x + \frac{1}{4} = 3 + \frac{1}{4}$$

$$(x - \frac{1}{2})^2 = 3\frac{1}{4} = \frac{13}{4}$$

$$x - \frac{1}{2} = \sqrt{\frac{13}{4}} = \frac{1}{2}\sqrt{13} \quad \text{of} \quad x - \frac{1}{2} = -\frac{1}{2}\sqrt{13}$$

$$x = \frac{1}{2} + \frac{1}{2}\sqrt{13} \quad \text{of} \quad x = \frac{1}{2} - \frac{1}{2}\sqrt{13}$$

$$x^2 - 9x - 3 = 0$$

$$x^2 - 9x - 3 + 23\frac{1}{4} = 23\frac{1}{4}$$

$$x^2 - 9x + 20\frac{1}{4} = 23\frac{1}{4}$$

$$(x - 4\frac{1}{2})^2 = 23\frac{1}{4} = \frac{93}{4}$$

$$x - 4\frac{1}{2} = \sqrt{\frac{93}{4}} = \frac{1}{2}\sqrt{93} \quad \text{of} \quad x - 4\frac{1}{2} = -\frac{1}{2}\sqrt{93}$$

$$x = 4\frac{1}{2} + \frac{1}{2}\sqrt{93} \quad \text{of} \quad x = 4\frac{1}{2} - \frac{1}{2}\sqrt{93}$$

34 a $a = 2$, $b = 12$ en $c = 6$

b

$$x = \frac{-12 + \sqrt{12^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2} = \frac{-12 + \sqrt{96}}{4} = \frac{-12 + 4\sqrt{6}}{4} = -3 + \sqrt{6}$$

$$x = \frac{-12 - \sqrt{12^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2} = \frac{-12 - \sqrt{96}}{4} = \frac{-12 - 4\sqrt{6}}{4} = -3 - \sqrt{6}$$

35 Dan staat er een lineaire vergelijking.

36 $2x^2 - 3x - 35 = 0$

$$\left. \begin{array}{l} a = 2 \\ b = -3 \\ c = -35 \end{array} \right\} D = 9 - 4 \cdot 2 \cdot -35 = 289, \sqrt{D} = 17$$

$$x = \frac{-3 + 17}{4} = 3\frac{1}{2} \quad \text{of} \quad x = \frac{-3 - 17}{4} = -5$$

$$2x^2 + 4x - 1 = 0$$

$$\left. \begin{array}{l} a = 2 \\ b = 4 \\ c = -1 \end{array} \right\} D = 16 - 4 \cdot 2 \cdot -1 = 24, \sqrt{D} = \sqrt{24} = 2\sqrt{6}$$

$$x = \frac{-4 + 2\sqrt{6}}{4} = -1 + \frac{1}{2}\sqrt{6} \quad \text{of} \quad x = \frac{-4 - 2\sqrt{6}}{4} = -1 - \frac{1}{2}\sqrt{6}$$

$$7x^2 - 6x + 2 = 0$$

$$\left. \begin{array}{l} a = 7 \\ b = -6 \\ c = 2 \end{array} \right\} D = 36 - 4 \cdot 7 \cdot 2 = -20$$

$D < 0$, dus géén oplossingen

$$\frac{1}{2}x^2 - 3x - 4\frac{1}{2} = 0$$

$$\left. \begin{array}{l} a = \frac{1}{2} \\ b = -3 \\ c = -4\frac{1}{2} \end{array} \right\} D = 9 - 4 \cdot \frac{1}{2} \cdot -4\frac{1}{2} = 18, \sqrt{D} = \sqrt{18} = 3\sqrt{2}$$

$$x = \frac{3 + 3\sqrt{2}}{1} = 3 + 3\sqrt{2} \quad \text{of} \quad x = \frac{3 - 3\sqrt{2}}{1} = 3 - 3\sqrt{2}$$

$$4x = 1 + 4x^2$$

$$4x^2 - 4x + 1 = 0$$

$$\left. \begin{array}{l} a = 4 \\ b = -4 \\ c = 1 \end{array} \right\} D = 16 - 4 \cdot 4 \cdot 1 = 0$$

$$x = -\frac{-4}{8} = \frac{1}{2}$$

$$(x-3)^2 = 5 - 3x$$

$$x^2 - 6x + 9 = 5 - 3x$$

$$x^2 - 3x + 4 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -3 \\ c = 4 \end{array} \right\} D = 9 - 4 \cdot 1 \cdot 4 = -7$$

$D < 0$, dus geen oplossingen

$$5x - 3x^2 = 0$$

$$\left. \begin{array}{l} a = -3 \\ b = 5 \\ c = 0 \end{array} \right\} D = 25 - 4 \cdot (-3) \cdot 0 = 25, \sqrt{D} = 5$$

$$x = \frac{-5+5}{-6} = 0 \quad \text{of} \quad x = \frac{-5-5}{-6} = \frac{10}{6} = 1\frac{2}{3}$$

OKER OPGAVEN

15 a $\frac{18}{45} = \frac{12}{y}$

$$18y = 540$$

$$y = 30$$

$$O = 12 \cdot (45 - 30) = 180$$

b $\frac{18}{45} = \frac{x}{y}$

$$18y = 45x$$

$$y = 2\frac{1}{2}x$$

c Breedte van de rechthoek is

$$45 - y = 45 - 2\frac{1}{2}x$$

$$O = x \cdot (45 - 2\frac{1}{2}x) = 45x - 2\frac{1}{2}x^2$$

d $45x - 2\frac{1}{2}x^2 = 0$

$$2\frac{1}{2}x(18 - x) = 0$$

$$x = 0 \quad \text{of} \quad x = 18$$

De oppervlakte is maximaal als $x = 9$.

e De oppervlakte is dan $45 \cdot 9 - 2\frac{1}{2} \cdot 9^2 = 202\frac{1}{2}$.

23 Als de top op de y-as ligt, dan zijn (-2,4) en (3,6) ook punten van de parabool. Dus dan moet het een dalparabool zijn.

29 a oppervlakte vierkant = $6 \cdot 6 = 36$
oppervlakte wit totaal = $x \cdot x + (6-x)(6-x) = 2x^2 - 12x + 36$

$$\text{oppervlakte oker} = 36 - (2x^2 - 12x + 36) = -2x^2 + 12x$$

b $-2x^2 + 12x = 0$

$$-2x(x-6) = 0$$

$$x = 0 \quad \text{of} \quad x = 6$$

De nulpunten zijn 0 en 6.

c Als $x = 3$, dan oppervlakte = $-2 \cdot 3^2 + 12 \cdot 3 = 18$.

30 a

$$\begin{cases} l \cdot b = 400 \\ 2l + 2b - 4 = 96 \end{cases}$$

b $2l + 2b - 4 = 96$

$$2l + 2b = 100$$

$$l + b = 50$$

$$l = 50 - b$$

$$(50 - b) \cdot b = 400$$

$$50b - b^2 = 400$$

$$b^2 - 50b + 400 = 0$$

$$(b-40)(b-10) = 0$$

$$b = 40 \quad \text{of} \quad b = 10$$

$$l = 10 \quad \quad \quad l = 40$$

De afmetingen zijn 40 bij 10 of 10 bij 40.

29.8 EXTRA OPGAVEN

1 a $y = x^2 + x$

Nulpunten :

$$x^2 + x = 0$$

$$x(x+1) = 0$$

$$x = 0 \quad \text{of} \quad x = -1$$

Snijpunt y - as :

$$y = 0^2 + 0 = 0$$

$$(0,0)$$

Symmetrie - as :

$$x = \frac{0-1}{2} = -\frac{1}{2}$$

$$y = \left(-\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$$

$$\text{Top}\left(-\frac{1}{2}, -\frac{1}{4}\right)$$

x	-3	-2	-1	$-\frac{1}{2}$	0	1	2
y	7	2	0	$-\frac{1}{4}$	0	2	7

$$y = x^2 - 7x$$

Nulpunten :

$$x^2 - 7x = 0$$

$$x(x-7) = 0$$

$$x = 0 \quad \text{of} \quad x = 7$$

Snijpunt y – as :

$$y = 0^2 - 7 \cdot 0 = 0$$

(0,0)

Symmetrie – as :

$$x = \frac{0+7}{2} = 3\frac{1}{2}$$

$$y = (3\frac{1}{2})^2 - 7 \cdot 3\frac{1}{2} = -12\frac{1}{4}$$

Top($3\frac{1}{2}, -12\frac{1}{4}$)

x	-1	0	1	2	3	$3\frac{1}{2}$	4	5	6	7
y	8	0	-6	-10	-12	$-12\frac{1}{4}$	-12	-10	-6	0

$$y = -3x^2$$

Nulpunten :

$$-3x^2 = 0$$

$$x = 0$$

Snijpunt y – as :

$$y = -3 \cdot 0^2$$

(0,0)

Symmetrie – as :

$$x = \frac{0+0}{2} = 0$$

$$y = -3 \cdot 0^2$$

Top(0,0)

x	-3	-2	-1	0	1	2
y	-27	-12	-3	0	-3	-12

$$y = (x+2)^2 - 3$$

Nulpunten :

$$(x+2)^2 - 3 = 0$$

$$(x+2)^2 = 3$$

$$x+2 = \sqrt{3} \quad \text{of} \quad x+2 = -\sqrt{3}$$

$$x = -2 + \sqrt{3} \quad \text{of} \quad x = -2 - \sqrt{3}$$

Snijpunt y – as :

$$y = (0+2)^2 - 3 = -3$$

(0,-3)

Top(-2,-3)

Symmetrie – as :

$$x = -2$$

x	-5	-4	-3	-2	-1	0	1
y	6	1	-2	-3	-2	1	6

$$y = -\frac{1}{2}(x-1)^2 + 8$$

Nulpunten :

$$-\frac{1}{2}(x-1)^2 + 8 = 0$$

$$-\frac{1}{2}(x-1)^2 = -8$$

$$(x-1)^2 = 16$$

$$x-1 = 4 \quad \text{of} \quad x-1 = -4$$

$$x = 5 \quad \text{of} \quad x = -3$$

Snijpunt y – as :

$$y = -\frac{1}{2}(0-1)^2 + 8 = 7\frac{1}{2}$$

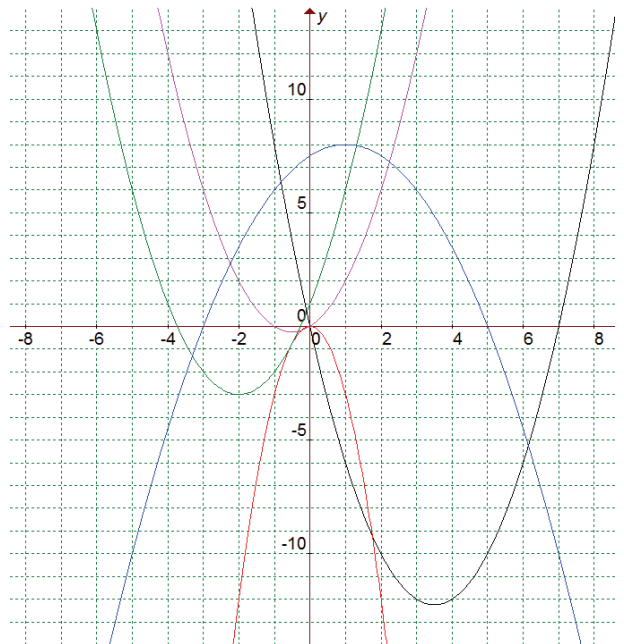
(0, $7\frac{1}{2}$)

Top(1,8)

Symmetrie – as :

$$x = \frac{-3+5}{2} = 1$$

x	-3	-2	-1	0	1	2	3	4	5	6
y	0	$3\frac{1}{2}$	6	$7\frac{1}{2}$	8	$7\frac{1}{2}$	6	$3\frac{1}{2}$	0	$-4\frac{1}{2}$



2 $y = x^2 + 12x$

Nulpunten :

$$x^2 + 12x = 0$$

$$x(x+12) = 0$$

$$x = 0 \quad \text{of} \quad x = -12$$

Symmetrie – as :

$$x = \frac{0-12}{2} = -6$$

$$y = (-6)^2 + 12 \cdot -6 = -36$$

Top(-6,-36)

$$y = 2x^2 - 5x$$

Nulpunten :

$$2x^2 - 5x = 0$$

$$2x(x - 2\frac{1}{2}) = 0$$

$$x = 0 \quad \text{of} \quad x = 2\frac{1}{2}$$

Symmetrie - as :

$$x = \frac{0+2\frac{1}{2}}{2} = 1\frac{1}{4}$$

$$y = 2 \cdot (1\frac{1}{4})^2 - 5 \cdot 1\frac{1}{4} = -3\frac{1}{8}$$

$$\text{Top}(1\frac{1}{4}, -3\frac{1}{8})$$

$$y = x^2 + 3x + 2$$

Nulpunten :

$$x^2 + 3x + 2 = 0$$

$$(x+1)(x+2) = 0$$

$$x = -1 \quad \text{of} \quad x = -2$$

Symmetrie - as :

$$x = \frac{-1-2}{2} = -1\frac{1}{2}$$

$$y = (-1\frac{1}{2})^2 + 3 \cdot -1\frac{1}{2} + 2 = -\frac{1}{4}$$

$$\text{Top}(-1\frac{1}{2}, -\frac{1}{4})$$

$$y = -x^2 + 4x + 6$$

$$-x^2 + 4x + 6 = 6$$

$$-x^2 + 4x = 0$$

$$-x(x-4) = 0$$

$$x = 0 \quad \text{of} \quad x = 4$$

Symmetrie - as :

$$x = \frac{0+4}{2} = 2$$

$$y = -2^2 + 4 \cdot 2 + 6 = 10$$

$$\text{Top}(2, 10)$$

3 a $y = cx^2$
 $3 = c \cdot 4^2$ (invullen het punt (4,3))
 $3 = 16c$

$$\frac{3}{16} = c$$

Vergelijking parabool: $y = \frac{3}{16}x^2$

b $x = 3$ of $x = -3 \Rightarrow y = \frac{3}{16} \cdot 3^2 = 1\frac{11}{16}$

Dus $(3, 1\frac{11}{16})$ en $(-3, 1\frac{11}{16})$

4 100 ; 10
 $4\frac{1}{2}$; $20\frac{1}{4}$; x
 $42\frac{1}{4}$; $6\frac{1}{2}$
5 ; $6\frac{1}{4}$; x
6 ; 36
1 ; 1

5 $14 = x(x-5)$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x = 7 \quad \text{of} \quad x = -2$$

$$2x^2 + x = 5x + 8$$

$$2x^2 - 4x - 8 = 0$$

$$x^2 - 2x - 4 = 0$$

$$x^2 - 2x - 4 + 5 = 5$$

$$x^2 - 2x + 1 = 5$$

$$(x-1)^2 = 5$$

$$x-1 = \sqrt{5} \quad \text{of} \quad x-1 = -\sqrt{5}$$

$$x = 1 + \sqrt{5} \quad \text{of} \quad x = 1 - \sqrt{5}$$

$$25 = 4(x+1)^2$$

$$(x+1)^2 = 6\frac{1}{4}$$

$$x+1 = 2\frac{1}{2} \quad \text{of} \quad x+1 = -2\frac{1}{2}$$

$$x = 1\frac{1}{2} \quad \text{of} \quad x = -3\frac{1}{2}$$

$$(x+1)^2 + (x+3)^2 = 4x^2$$

$$2x^2 + 8x + 10 = 4x^2$$

$$2x^2 - 8x - 10 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = 5 \quad \text{of} \quad x = -1$$

$$x^2 - 3x = 2x^2 + x + 1$$

$$x^2 + 4x + 1 = 0$$

$$x^2 + 4x + 1 + 3 = 3$$

$$x^2 + 4x + 4 = 3$$

$$(x+2)^2 = 3$$

$$x+2 = \sqrt{3} \quad \text{of} \quad x+2 = -\sqrt{3}$$

$$x = -2 + \sqrt{3} \quad \text{of} \quad x = -2 - \sqrt{3}$$

$$x^2 + 5x + 3 = 0$$

$$x^2 + 5x + 3 + 3\frac{1}{4} = 3\frac{1}{4}$$

$$x^2 + 5x + 6\frac{1}{4} = 3\frac{1}{4}$$

$$(x+2\frac{1}{2})^2 = 3\frac{1}{4} = \frac{13}{4}$$

$$x+2\frac{1}{2} = \sqrt{\frac{13}{4}} = \frac{1}{2}\sqrt{13} \quad \text{of} \quad x+2\frac{1}{2} = -\frac{1}{2}\sqrt{13}$$

$$x = -2\frac{1}{2} + \frac{1}{2}\sqrt{13} \quad \text{of} \quad x = -2\frac{1}{2} - \frac{1}{2}\sqrt{13}$$

6 a Oppervlakte grasveld = $4 \cdot 4 = 16$
 Oppervlakte border = $3 \cdot 4x + 2 \cdot x^2 = 12x + 2x^2$
 $12x + 2x^2 = 16$

$$x^2 + 6x - 8 = 0$$

$$x^2 + 6x - 8 + 17 = 17$$

$$x^2 + 6x + 9 = 17$$

$$(x+3)^2 = 17$$

$$x+3 = \sqrt{17} \quad \text{of} \quad x+3 = -\sqrt{17}$$

$$x = -3 + \sqrt{17} \quad \text{of} \quad x = -3 - \sqrt{17}$$

Dus $x = -3 + \sqrt{17}$ m

b $12x + 2x^2 = 2 \cdot 16$

$$x^2 + 6x - 16 = 0$$

$$(x+8)(x-2) = 0$$

$$x = -8 \quad \text{of} \quad x = 2$$

Dus $x = 2$ m

7 $3x^2 + 10x + 3 = 0$

$$\left. \begin{array}{l} a = 3 \\ b = 10 \\ c = 3 \end{array} \right\} D = 100 - 4 \cdot 3 \cdot 3 = 64, \sqrt{D} = 8$$

$$x = \frac{-10+8}{6} = -\frac{1}{3} \quad \text{of} \quad x = \frac{-10-8}{6} = -3$$

$$2x^2 = 5x - 3$$

$$2x^2 - 5x + 3 = 0$$

$$\left. \begin{array}{l} a = 2 \\ b = -5 \\ c = 3 \end{array} \right\} D = 25 - 4 \cdot 2 \cdot 3 = 1, \sqrt{D} = 1$$

$$x = \frac{5+1}{4} = 1\frac{1}{2} \quad \text{of} \quad x = \frac{5-1}{2} = 2$$

$$x^2 - 8x = -22$$

$$x^2 - 8x + 22 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -8 \\ c = 22 \end{array} \right\} D = 64 - 4 \cdot 1 \cdot 22 = -24$$

$D < 0$, dus geen oplossingen

$$-5x^2 + 4x - \frac{4}{5} = 0$$

$$\left. \begin{array}{l} a = -5 \\ b = 4 \\ c = -\frac{4}{5} \end{array} \right\} D = 16 - 4 \cdot (-5) \cdot (-\frac{4}{5}) = 0$$

$$x = -\frac{4}{-10} = \frac{2}{5}$$

8 a oppervlakte driehoek = $\frac{1}{2} \cdot x(8-x) = 4x - \frac{1}{2}x^2$

b $\frac{1}{4}$ deel ; $\frac{1}{4} \cdot 8 \cdot 8 = 16$

c $2 \cdot x(8-x) = 16$

$$2x^2 - 16x + 16 = 0$$

$$x^2 - 8x + 8 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -8 \\ c = 8 \end{array} \right\} D = 64 - 4 \cdot 1 \cdot 8 = 32, \sqrt{D} = \sqrt{32} = 4\sqrt{2}$$

$$x = \frac{8+4\sqrt{2}}{2} = 4+2\sqrt{2} \text{ cm of } x = 4-2\sqrt{2} \text{ cm}$$

9 a hoogte = h , breedte = $h+3$, lengte = $h+4$

b oppervlakte =

$$2(h(h+4) + h(h+3) + (h+4)(h+3)) =$$

$$2(3h^2 + 14h + 12) = 6h^2 + 28h + 24$$

$$6h^2 + 28h + 24 = 162$$

$$6h^2 + 28h - 138 = 0$$

$$\left. \begin{array}{l} a = 6 \\ b = 28 \\ c = -138 \end{array} \right\} D = 784 + 3312 = 4096, \sqrt{D} = 64$$

$$x = \frac{-28+64}{12} = 3 \quad \text{of} \quad x = \frac{-28-64}{12} = -7\frac{2}{3}$$

Alleen $x = 3$ voldoet.

10 a $50t - 5t^2 = 0$

$$5t(10-t) = 0$$

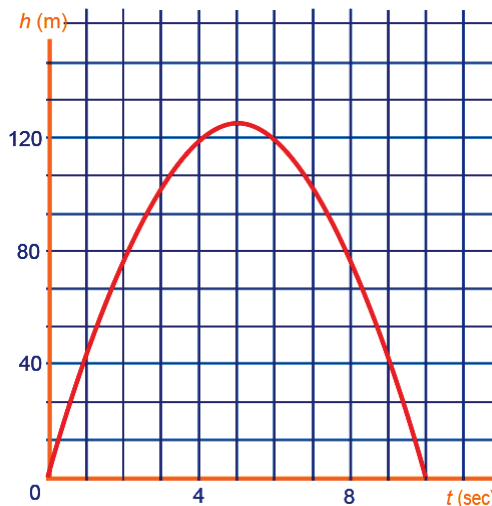
$$t = 0 \quad \text{of} \quad t = 10$$

Dus de vlucht duurt $10 - 0 = 10$ sec.

b maximale hoogte wordt bereikt na 5 sec.

$$h = 50 \cdot 5 - 5 \cdot 5^2 = 250 - 125 = 125 \text{ m}$$

c



d $50t - 5t^2 > 113,75$

$$0 > 5t^2 - 50t + 113,75$$

$$t^2 - 10t + 22,75 < 0$$

$$(t-3,5)(t-6,5) < 0$$

$$3,5 < t < 6,5$$

Dus tussen de 3,5 en 6,5 sec. is de hoogte van de vuurpijl meer dan 113,75 m.