17 De stelling van Pythagoras

80 · 125 = 10 000 cm²	5000 cm²

A: $\frac{1}{2} \cdot 6 \cdot 4 = 12$ cm²

B: 8 cm²

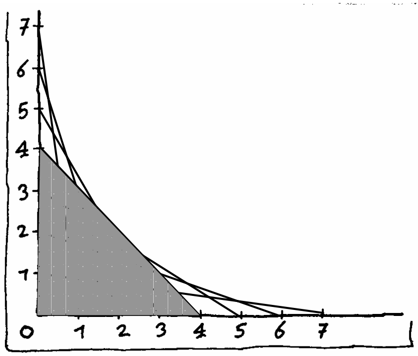
C: 6 cm²

D: 9 cm²

E: 15 cm²

F: $7\frac{1}{2}$ cm²

$600 - \frac{1}{2} \cdot 10 \cdot 5 - \frac{1}{2} \cdot 10 \cdot 15 - \frac{1}{2} \cdot 10 \cdot 5 - \frac{1}{2} \cdot 10 \cdot 10 = 425$ m²



$\frac{1}{2} \cdot 4 \cdot 4 = 8$ cm²

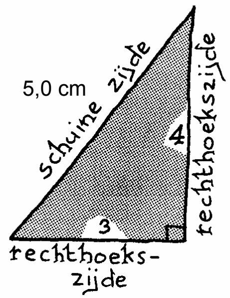
A: $6^2 - \frac{1}{2} \cdot 5 \cdot 1 \cdot 4 = 26$ m²

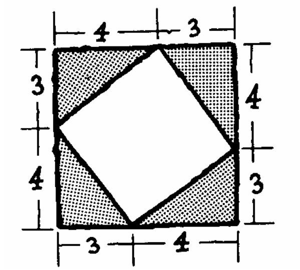
B $6^2 - \frac{1}{2} \cdot 4 \cdot 2 \cdot 4 = 20$ m²

C $6^2 - \frac{1}{2} \cdot 4 \cdot 3 \cdot 4 = 18$ m²

D 20 m²

E 26 m²

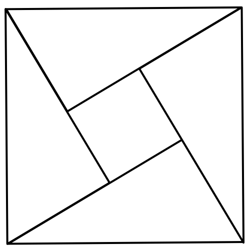




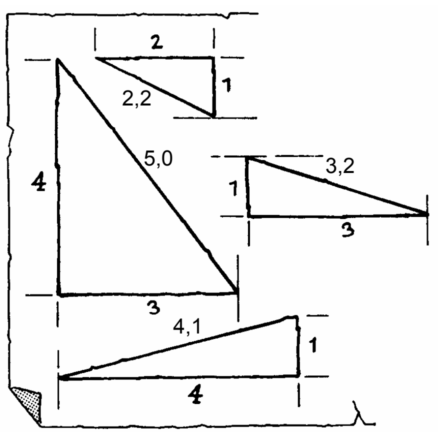
$4 \cdot \frac{1}{2} \cdot \cdot 4 \cdot 3 = 24$

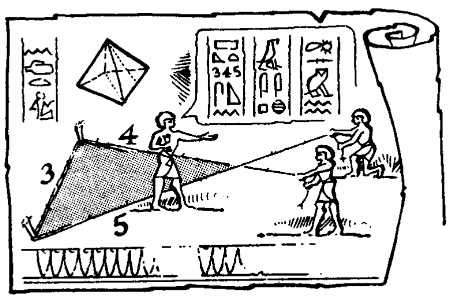
$49 - 24 = 25$

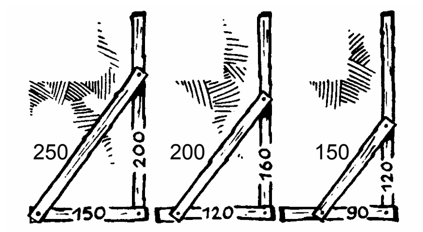
Een vierkant met oppervlakte 25 heeft zijden van lengte 5.

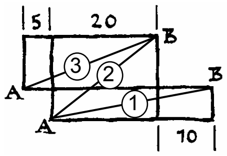


Voor de tweede puzzel:
zie bladzijde 9







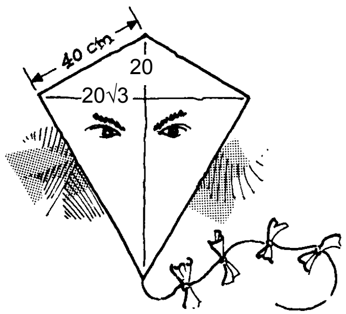


① $\sqrt{925}$

② $\sqrt{625}$

③ $\sqrt{725}$

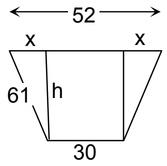
Dus ② is het kortst.



de helft is $40 \cdot \frac{1}{2}\sqrt{3}$, dus $40\sqrt{3}$

$40\sqrt{3}$, want de driehoek onder korte diagonaal is gelijkzijdig

oppervlakte $= \frac{1}{2} \cdot 80 \cdot 40\sqrt{3} = 1600\sqrt{3}$



$x = \frac{1}{2} \cdot (52 - 30) = 11$

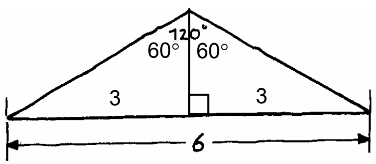
$h^2 = 61^2 - 11^2 = 3600$,

dus $h = 60$

$a^2 = 7\frac{1}{2}^2 + 30^2 = 9056\frac{1}{4}$

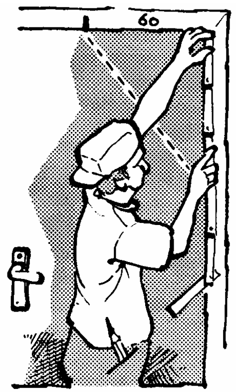
$a = 30{,}92$ cm

$b = \pi \cdot 15 \approx 47{,}12$ cm



$BC = 3{:}\sqrt{3}$

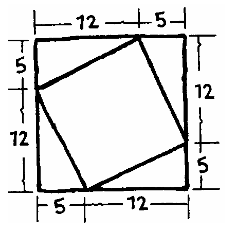
oppervlakte $= 3 \cdot 3{:}\sqrt{3}$ ($\approx 5{,}2$)



100 cm

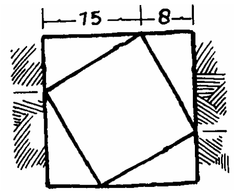
kleiner dan 90°

meer dan 100 cm



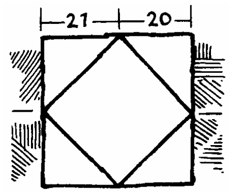
$289 - 2 \cdot 5 \cdot 12 = 169$

13 want $13 \cdot 13 = 169$



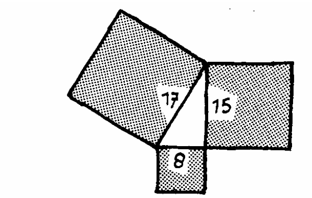
$23 \cdot 23 - 2 \cdot 8 \cdot 15 = 289$

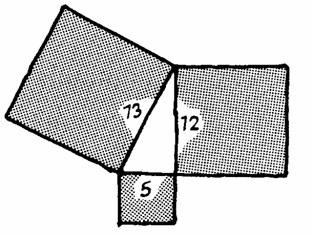
17, want $17 \cdot 17 = 289$



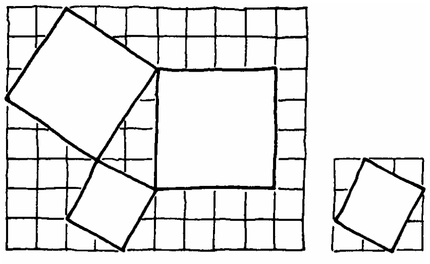
vierkant in het midden heeft oppervlakte $41 \cdot 41 - 4 \cdot \frac{1}{2} \cdot 20 \cdot 21 = 841$, dus de zijde is 29.

$2 \cdot 29 = 58$



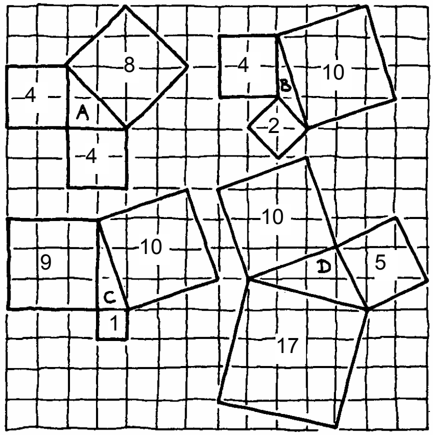


	1	2
3-4-5-driehoek	*9 + 16*	*25*
5-12-13-driehoek	25 + 144	169
8-15-17-driehoek	64 + 225	289
20-21-29-driehoek	400 + 441	841

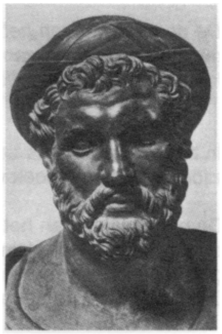


$3 \cdot 3 - 4 \cdot 2 \cdot \frac{1}{2} \cdot 1 = 5$ klopt

$5 \cdot 5 - 4 \cdot \frac{1}{2} \cdot 2 \cdot 3 = 13$; $16 \neq 13 + 5$



A en C



borstbeeld van (waarschijnlijk) Pyt-ha-goras, Grieks filo-soof (6e eeuw v. Chr.)

$AC^2 = 6^2 + 8^2 = 100$, dus $AC = 10$

$DB^2 = 17^2 - 8^2 = 225$, dus $DB = 15$

$AB = 21$

$21^2 = 441$; $10^2 + 17^2 = 389$; $441 > 389$, dus stomp

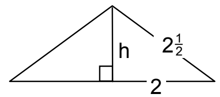
$AB^2 = 3^2 + 1^2 = 10$, dus $AB = \sqrt{10}$

$AC^2 = 6^2 + 2^2 = 40$, dus $AC = \sqrt{40}$

$BC^2 = 7^2 + 1^2 = 50$, dus $BC = \sqrt{50}$

Er geldt: $AB^2 + AC^2 = BC^2$, dus recht

$\sqrt{18^2 + 13^2 + 6^2} = 23$ cm
Past dus niet.

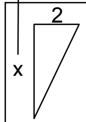


$h^2 = 2\frac{1}{2}^2 - 2^2 = 2\frac{1}{4}$

Dus $h = 1\frac{1}{2}$

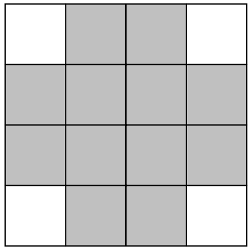
oppervlakte $= 1\frac{1}{2} \cdot 2 = 3$

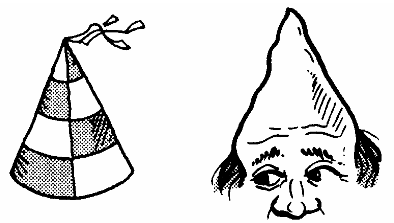
$\frac{1}{2} \cdot 4{,}8 \cdot 4 = 9{,}6\ m^2$



Links is een dakvlak getekend, x is de schuine kant van de voorgevel, dus $x^2 = 2^2 + 4{,}8^2 = 27{,}04$, $x = 5{,}2$.
oppervlakte dakvlak $= \frac{1}{2} \cdot 5{,}2 \cdot 2 = 5{,}2$.
Oppervlakte dak $= 8 \cdot 5{,}2 = 41{,}6\ m^2$

Dakgoot is schuine zijde van dakvlak:
$x^2 + 2^2 = 31{,}04$, dus goot is $\sqrt{31{,}04} \approx 5{,}6$ m



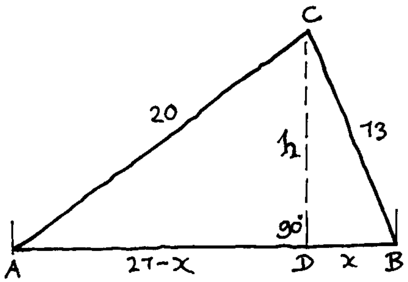


$\frac{1}{3} \cdot 2\pi \cdot 27 \approx 56$ cm

$\frac{1}{3} \cdot 2\pi \cdot 27 : (2\pi) = 9$ is de straal.

hoogte$^2 = 27^2 - 9^2 = 648$

hoogte $\approx 25{,}46$ cm



Dat is de stelling van Pythagoras in driehoek ACD.

$h^2 = 13^2 - x^2$
(stelling van Pythagoras in driehoek DBC)

$13^2 - x^2 = 400 - (21 - x)^2$

$169 - x^2 = 400 - (441 - 42x + x^2)$

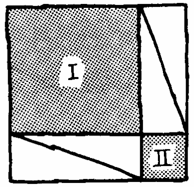
$169 - x^2 = -41 + 42x - x^2$

$42x = 210$

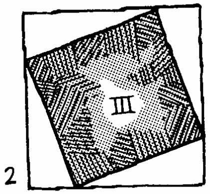
$x = 5$

$h^2 = 13^2 - 5^2 = 144$, dus $h = 12$

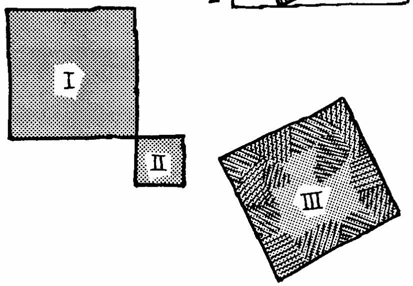
oppervlakte $= \frac{1}{2} \cdot 12 \cdot 5 + \frac{1}{2} \cdot 12 \cdot 16 = 126$

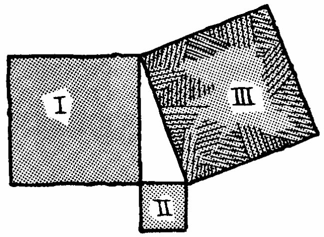


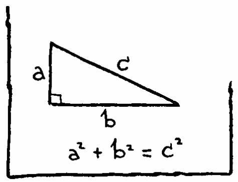
1

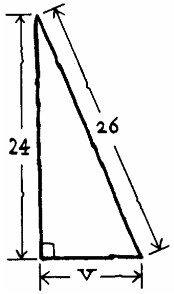


2





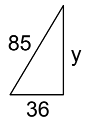




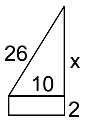
Berekening x:
$16^2 + x^2 = 34^2$
$x^2 = 900$
$x = 30$

Berekening y:
$y^2 + 60^2 = 61^2$
$y^2 = 121$
$y = 11$

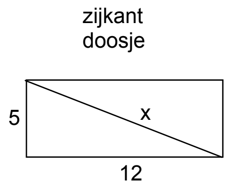
$x^2 = 84^2 + 13^2 = 7225$
$x = 85$ dm



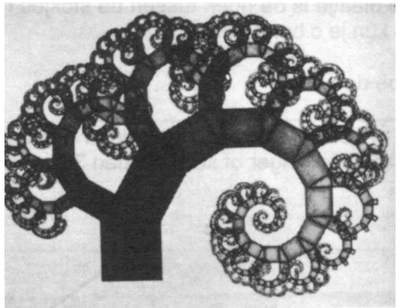
$y^2 + 36^2 = 85^2$
$y^2 = 5929$
$y = 77$ dm



$x^2 + 10^2 = 26^2$
$x^2 = 576$
$x = 24$ hoogte $= 24 + 2 = 26$ m



$x^2 = 5^2 + 12^2 = 169$, dus $x = 13$.
De foto is 13 bij 18 cm

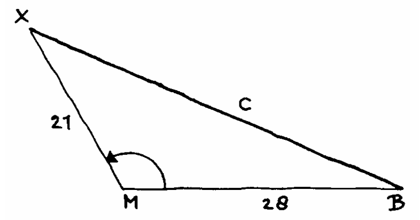


2

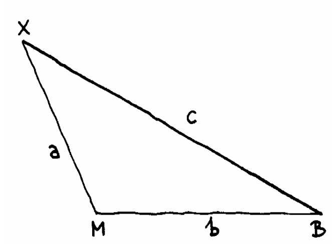
6

3 5 6

12 3



$c^2 = 21^2 + 28^2 = 1225$, dus $c = 35$



Voor het linker plaatje: $a^2 + b^2 > c^2$

Voor het rechter plaatje: $a^2 + b^2 < c^2$

$AB^2 = 7^2 + 1^2 = 50$, dus $AB = \sqrt{50}$

$AC^2 = 6^2 + 3^2 = 45$, dus $AC = \sqrt{45}$

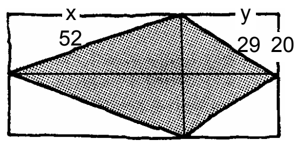
$AD^2 = 5^2 + 4^2 = 41$, dus $AD = \sqrt{41}$

$AE^2 = 5^2 + 5^2 = 50$, dus $AE = \sqrt{50}$

$AF^2 = 4^2 + 6^2 = 52$, dus $AF = \sqrt{52}$

Geldt: $AB^2 = AC^2 + BC^2$?
$50 = 45 + 5$? Ja, dus hoek ACB is recht.

Zie plaatje hieronder.
$x^2 = 52^2 - 20^2 = 2304$, dus $x = 48$
$y^2 = 29^2 - 20^2 = 441$, dus $y = 21$
Dus $x + y = 69$ cm



$\frac{1}{2} \cdot 40 \cdot 69 = 1380$ cm^2

$\angle ACB = 180° - 30° - 105° = 45°$

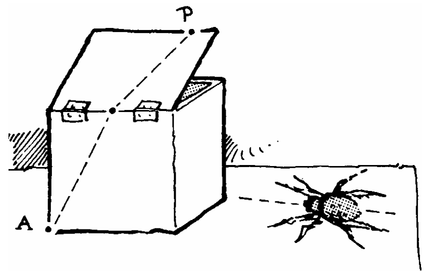
$BC = 2\sqrt{2}$ (driehoek BCD is een half vierkant)
$DC = 2$
$AD = 2 \cdot \sqrt{3} = 2\sqrt{3}$ en $AB = 2 \cdot 2 = 4$
(Driehoek ABD is een halve gelijkzijdige driehoek.)
Dus $AB = 4$, $BC \approx 2{,}8$ en $AC \approx 5{,}5$

Ada: $\sqrt{200} + \sqrt{1300}$

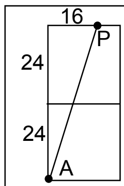
Bart: $\sqrt{500} + \sqrt{800}$

route 2 – route 1 $\approx$ 4 dm

$AB = \sqrt{(30^2+40^2)} = 50$ m



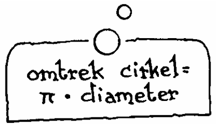
Nee

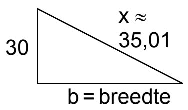


$AP^2 = 16^2 + 48^2 = 2560$

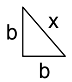
Dus $AP = \sqrt{2560} \approx 50{,}6$ cm

$\pi \cdot x = 110$, dus $x = 110 : \pi \approx 35{,}01$ cm





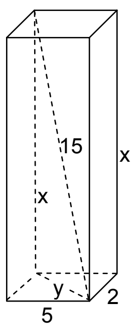
$b^2 + 30^2 = x^2$, dus $b^2 = 325{,}7001$
$b = \sqrt{325{,}7001} \approx 18$ cm



$b^2 + b^2 = x^2 = 1225$

$b^2 = 1225 : 2 = 612{,}5$

$b = \sqrt{612{,}5} \approx 25$ cm



$y^2 = 2^2 + 5^2 = 29$

$x^2 + y^2 = 15^2$, dus $x^2 + 29 = 225$

$x = \sqrt{196} = 14$ cm

$BC^2 = 15^2 - 9^2 = 144$, dus $BC = \sqrt{144} = 12$

$BD^2 = 20^2 - 12^2 = 256$, dus $BD = \sqrt{256} = 16$

$AD = 25$, dus $AD^2 = AC^2 + CD^2$, dus hoek C is recht.

De zijden van driehoek ABC zijn 9, 12 en 15. De zijden van driehoek ACD zijn $1\frac{2}{3}$ keer zo groot, dus de driehoeken ABC en ACD zijn gelijkvormig.

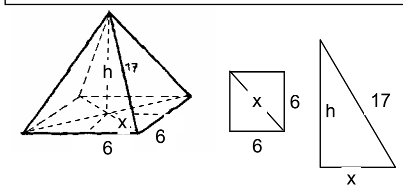
fig 1: $x^2 = 18^2 - 17^2 = 35$, dus $x = \sqrt{35}$

$y^2 = 19^2 - 17^2 = 72$, dus $y = \sqrt{72}$

fig 2: $x^2 = 22^2 - 20^2 = 84$, dus $x = \sqrt{84}$
$z = x + y$, dan $z^2 = 25^2 - 20^2 = 225$
$x + y = \sqrt{225} = 15$, dus
$y = 15 - \sqrt{84} \approx 5{,}8$

fig 3: $x^2 = 2^2 + 3^2 = 13$,

$y^2 = 6^2 + x^2 = 49$, dus $y = 7$



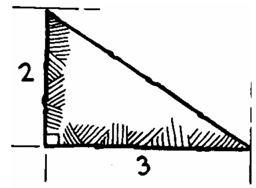
$x^2 = 6^2 + 6^2 = 72$

$h^2 + x^2 = 17^2$, dus $h^2 + 72 = 289$.

dus $h = \sqrt{217} \approx 14{,}7$

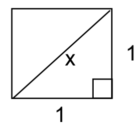
$7^2 + 4^2 = 65 > 8^2$;
P is dus een scherpe hoek.
De driehoek is scherphoekig.

$30^2 + 16^2 = 1156$
$34^2 = 1156$, dus hoek A is recht.
De driehoek is rechthoekig

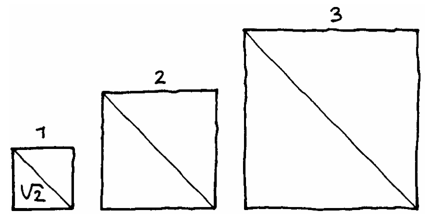


$2^2 + 3^2 = 13$

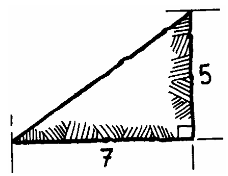
langer want $12{,}96 < 13$



$1^2 + 1^2 = 2 = x^2$ $x = \sqrt{2}$



$2\sqrt{2}$	$3\sqrt{2}$



$7^2 + 5^2 = 74$, dus $\sqrt{74} \approx 8{,}60$

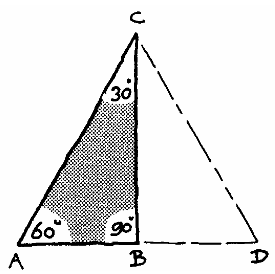
$y^2 = 14^2 - 10^2 = 96$, dus $y = \sqrt{96} \approx 9,80$

$z^2 = y^2 + 2^2 = 96 + 4 = 100$, dus $z = 10$

$x^2 = 12^2 - 9^2 = 63$, dus $x = \sqrt{63}$
$y^2 = 14^2 - 9^2 = 115$, dus $y = \sqrt{115}$
$x + y = \sqrt{63} + \sqrt{115} \approx 18,7$

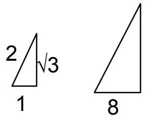
$a^2 = 1^2 + 3^2 = 10$, dus $a = \sqrt{10}$
$b^2 = a^2 + 1 = 11$, dus $b = \sqrt{11}$
$c^2 = b^2 + 1 = 12$, dus $c = \sqrt{12}$
$d^2 = c^2 + 1 = 13$, dus $d = \sqrt{13}$

$\sqrt{3} \cdot \sqrt{3}$	=	3
$2 \cdot \sqrt{3} \cdot \sqrt{3}$	=	$2 \cdot 3 = 6$
$(2 \cdot \sqrt{3})^2$	=	$2 \cdot 2 \cdot \sqrt{3} \cdot \sqrt{3} = 2 \cdot 2 \cdot 3 = 12$
$\sqrt{(\sqrt{16})}$	=	$\sqrt{4} = 2$
$\sqrt{(168^2)}$	=	168
$(\sqrt{168})^2$	=	168

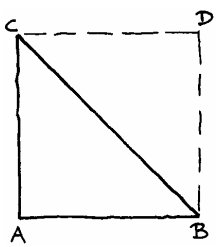


1 (de helft vanwege symmetrie)

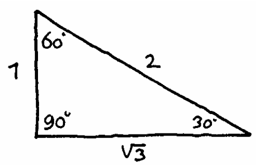
$\sqrt{2^2 - 1^2} = \sqrt{3}$

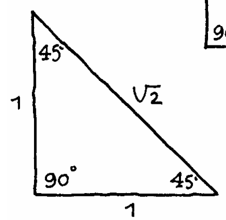


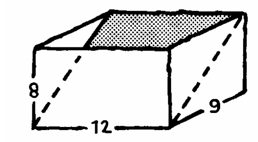
De tweede driehoek is de eerste uitvergroot met factor 8, de zijden zijn dus: 8, 16 en $8\sqrt{3}$.



$4\sqrt{2}$



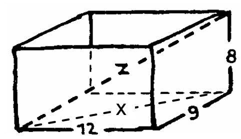




$12^2 + 9^2 = 225$, dus links: 8 bij $\sqrt{225} = 15$

$12^2 + 8^2 = 208$, dus midden: 9 bij $\sqrt{208}$

$9^2 + 8^2 = 145$, dus rechts: 12 bij $\sqrt{145}$



$x^2 = 12^2 + 9^2 = 144 + 81 = 225$

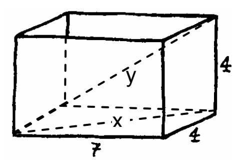
$z^2 = 8^2 + x^2 = 64 + 225 = 289$

$z = \sqrt{289} = 17$

$y^2 = 8^2 + 9^2 = 145$

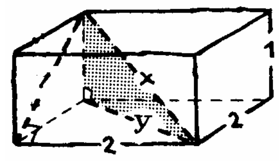
$z^2 = 12^2 + y^2 = 289$

$z = \sqrt{289} = 17$



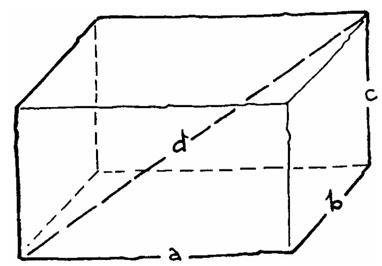
$x^2 = 4^2 + 7^2 = 65$

$y^2 = x^2 + 4^2 = 81$, dus $y = 9$



$y^2 = 2^2 + 2^2 = 8$
dus $y = \sqrt{8} \approx 2,8$
dus $x^2 \approx 1^2 + 2,8^2 = 8,84$
dus $x \approx \sqrt{8,84} \approx 2,97$ dm

Er wordt tussentijds twee keer afgerond!
$y^2 = 2^2 + 2^2 = 8$
$x^2 = y^2 + 1 = 9$
$y = \sqrt{9} = 3$ precies!



$\sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13$ dm

$x^2 = 6^2 + 6^2 + 7^2 = 121$, dus $x = 11$